Name of the course : **CBCS B.Sc(H)Mathematics**

Unique Paper Code : **32357609**

Name of Paper : **DSE-3 : Bio-Mathematics**

Semester : **VI**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Describe the mathematical model of population growth for any species changing by birth only. Draw and describe the graph of the behavior of the population with an increase in time. What would be the change in your model if you consider deaths of individuals also along with births. In a research on population dynamics of mosquitoes, it was estimated that the initial population is 2000. Over the time period of one month, 300 births and 100 deaths were recorded in the population. Predict the population size at the end of 10 months.

2. Find and draw the trajectories in the phase plane of

a) $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0.$

b) $\frac{dx}{dt} = 5x + 2y\,;\ \frac{dy}{dt} = -2x + 5y\,.$

3. What is the threshold effect in a heartbeat model? Why is it an important feature to be included in the model? Modify the following model for heartbeat to reflect the threshold effect.

$$\epsilon \frac{dx}{dt} = -a\,(x - x_0) - (b - b_0)$$

$$\frac{db}{dt} = x - x_0$$

where (b_0, x_0) is the unique rest state. Present a phase plane analysis of the model obtained above and explain how it includes the physiological considerations of the heartbeat cycle?

4. Define Hopf bifurcation. Show that the system

$$x' = -y + x(\beta - x^2 - y^2)$$

$$y' = x + y(\beta - x^2 - y^2)$$

has Hopf bifurcation. Consider the iteration scheme for points on a Poincare Plane:

$$x_{n+1} = \frac{1}{2} y_n$$

$$y_{n+1} = -x_n + \frac{1}{2}\alpha y_n - y_n^3$$

Show that there is a bifurcation at $\alpha = 3$. Show that the limit cycle in $0 < \alpha < 3$is stable and becomes of saddle type when αexceeds 3.

5. Consider ancestral and descendent sequences of 400 bases which were simulated according to Kimura 2-parameter model with $\gamma = \frac{\beta}{5}$.A comparison of aligned sites gave the frequency data in Table-1 below:

Table -1

S_1/S_0	A	G	C	T
A	92	15	2	2
G	13	84	4	4
C	0	1	77	16
T	4	2	14	70

Compute the Jukes-Cantor distance and Kimura 2-parameter distance to 10 decimal digits.

In what cases Jukes-Cantor distance is zero? Derive the formula $d_{JC} = -\frac{3}{4}ln(\frac{4q-1}{3})$ where q is the proportion of bases that are the same in the before and after sequences.

6. Draw the single topologically distinct unrooted bifurcating tree that could describe the relationship between 3 taxa. Draw Punnett square for $Dd\ X\ Dd$ and $DdWw\ X\ ddWw$, where D denotes allele for dominant plant, d allele for dwarf plant, W dominant allele for round seeds and w recessive allele for wrinkled seeds. What is the genotypic ratio and phenotypic ratio in $Dd\ X\ Dd$? What percentage is the progeny of $DdWw\ X\ ddWw$ dwarf with round seeds? Now consider the distance data in the following Table-2, which is exactly fit by the following tree, use UPGMA to construct a tree from this data. Also use Neighbor-Joining method to compute R_1 , R_2 , R_3 and R_4 , and then a table of values for M for the taxa $S1$, $S2$, $S3$ and $S4$.

Table-2

	$S1$	$S2$	$S3$	$S4$
$S1$		0.3	0.4	0.5
$S2$			0.5	0.4
$S3$				0.7